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Relaxation of kinks in the Ising chain with a transverse field interacting with a three-dimensional phonon field

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Abstract. The influence of phonon fluctuations on the dynamics of kink-like domain-wall excitations in the one-dimensional Ising model with a transverse field ($s = \frac{1}{2}$) was investigated. It was found that such non-linear excitations behave almost like classical Brownian particles. This is the consequence of the emission and absorption of acoustic phonons excited by domain walls propagating through the vibrating lattice. The effective friction and diffusion constants are also evaluated. It was established that the overall process has the character of Cherenkov-type radiation.

1. Introduction

The physical properties of various substances can be affected a great deal by the presence of pulse- or kink-like excitations, known as solitons. Owing to their extreme stability, even in non-integrable models as mainly adopted for 3D systems, the kink solitons play an important role in understanding a number of phenomena in real physical systems. They thus represent a fairly good description of real objects such as domain walls (DW) in magnetics or ferroelectrics [1, 2]. Furthermore the appearance of a central peak in the energy spectrum of slow neutrons scattered by quasi-1D ferromagnets, for example $CsNiF_3$ [3, 4], or by some uniaxial ferroelectrics (undergoing a quasi-1D structural phase transition (SPT)) [5, 6] was quite well explained on the basis of the soliton concept.

The theoretical description of kinks is based upon the application of several rather simple 1D models such as sine–Gordon, so-called φ-four (φ^4), Ising model with a transverse field (IMTF), etc. Such a simplified treatment is not satisfactory for the analysis of kink properties in some realistic situations when various (external or internal) perturbations can affect their dynamics significantly. Consequently there is an increasing interest in the investigation of soliton behaviour in more realistic models. In that context the problem is mainly reduced to the study of both the time evolution of the soliton parameters and the corresponding changes of the soliton shape [7]. Furthermore, for a consistent treatment of both the kinetics and the thermodynamic properties of kinks in the presence of perturbative fields, an adequate kinetic equation (KE) has to be constituted [8].

The investigation of the kinetics of kinks in IMTF is especially important owing to their broad application in the theory of magnetism [9, 10], SPT [11, 12], the theory of Frenkel excitons [13], etc. One type of perturbation is always present in real spin systems

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even in an ideal lattice. These are the vibrations of the crystal lattice, which considerably affect the kink dynamics depending on the strength of spin–phonon interaction.

In almost all previous approaches to the problem of kink–phonon interaction both (spin and vibrational) subsystems were considered as purely one-dimensional ($d = 1$) [14–19]. However, although the spin subsystem could exhibit large anisotropy and could be regarded as 1D, this is not necessarily the case with the vibrational one. Thus, for example, the ferromagnetic $CoCl_2 \cdot 2NC_5H_5$ is composed of a collection of linear chains, where $CoCl_2$ units are aggregated into the 1D Ising-like ferromagnetic structure embedded into the monoclinic crystal lattice [20]. To this particular example one could add the whole family of 1D Ising ferromagnets that belong to the series of isomorphic transition-metal halides: $AMB_3 \cdot 2aq$ (A = Cs, Rb; M = Mn, Co, Fe; B = Cl, Br; aq = H_2O or D_2O) [21]. In addition some uniaxial ferroelectric materials (CsH_2PO_4 or $Pb_5Ge_3O_{11}$, for example) also belong to the class of realistic systems in which 1D Ising-like subsystems exist within 3D crystal lattices [6, 22].

It is obvious that the theoretical description of kinks in real quasi-1D substances must be based on more appropriate models where these differences in dimensionalities of spin and phonon subsystems could be taken into account explicitly. For that purpose we shall utilize the continuum counterpart of the model introduced by Pytte [23], who studied the system of non-interacting Ising chains stacked in a 3D phonon field. This model was later generalized by Mijatović and Milošević [24], who considered the magnetic phase transitions in the system of weakly coupled Ising chains.

2. Model

The Hamiltonian of the system we deal with has the form

$$H = H_s + H_{int} + H_{ph}. \tag{1}$$

Here H_s is the Hamiltonian of the IMTF, which in the continuum approximation reads [25]

$$H_s = -\Omega s \int \frac{dx}{R_0} \cos\varphi(x,t)\,[1 - u^2(x,t)]^{1/2} - Js^2 \int \frac{dx}{R_0}\,[u^2(x,t) - \tfrac{1}{2}R_0^2\,u_x^2(x,t)] \tag{2}$$

where $R_0 = |\boldsymbol{R}_0|$ is the lattice constant along the chain, $u(x,t) = \cos\theta(x,t)$, $\varphi(x,t)$ and $\theta(x,t)$ are polar angles of the spin (or pseudospin) vector, while Ω and J represent the energy parameters of the transverse field (actual magnetic or effective, tunnelling-like) and the nearest-neighbour longitudinal coupling, respectively.

The interaction between the spin subsystem and the longitudinal (acoustic only) phonons is defined by the following Hamiltonian;

$$H_{int} = -\frac{s^2}{\sqrt{N}} \sum_q \int \frac{dx}{R_0} F_q\, e^{iq\cdot x}(b_q + b^+_{-q})[u^2(x,t) - \tfrac{1}{2}R_0^2\,u_x^2(x,t)] \tag{3}$$

where

$$F_q = \alpha\left(\frac{\hbar}{2m\omega_q}\right)^{1/2}(e^{iq\cdot R_0} - 1)\frac{(\boldsymbol{e}_q \cdot \boldsymbol{R}_0)}{|\boldsymbol{e}_q|\,|\boldsymbol{R}_0|}$$

is the Fourier component of the spin(pseudospin)–phonon interaction induced by the coupling-strength parameter $\alpha = (\partial J/\partial x)|_{x=x_0}$, m is the mass of each magnetic (or ferroelectrically active) ion and $\boldsymbol{e}_q$ is the polarization vector of longitudinal acoustic phonons with frequency ω_q.

The phonon Hamiltonian has the standard form:

$$H_{\mathrm{ph}} = \sum_q \hbar\omega_q b_q^+ b_q. \tag{4}$$

3. Unperturbed domain walls

In the absence of perturbations all main features of the IMTF depend merely on a single parameter $\lambda = J/2\Omega$. In the ground state (GS) all spins are aligned, i.e. $\varphi(x) = 0$, $\theta(x) = \theta_0 = \sin^{-1}(1/2\lambda)$. The most interesting regime is one when the GS exhibits spontaneous symmetry breaking (spontaneous polarization in S_z). This can happen only if λ exceeds some critical value, $\lambda > \lambda_c = 1/2$. Then the class of lowest-energy excitations above the GS represents a single domain wall (DW), i.e. a polarization kink. These excitations correspond to a partial solution (transcendental in general [25]) of equation (2) with the boundary conditions $\varphi(x \to \mp\infty) = 0$, $u(x \to \mp\infty = \mp\cos\theta_0$, thus breaking the long-range order in S_z.

The dyamics of the system is governed by the pair of Landau–Lifshitz equations where $u(x, t)$ and $\varphi(x, t)$ play the role of generalized momentum and coordinate:

$$\dot{u} = -\frac{1}{\hbar s}\frac{\delta \mathcal{H}_s}{\delta\varphi} \qquad \dot{\varphi} = \frac{1}{\hbar s}\frac{\delta\mathcal{H}_s}{\delta u} \tag{5}$$

where, with account of equation (2), $\mathcal{H}_s$ is the Hamiltonian density of the unperturbed spin subsystem. The corresponding Lagrangian density associated with the above equations has the convenient form

$$L_s(x, t) = \hbar s u(x, t)\, \partial\varphi/\partial t - \mathcal{H}_s. \tag{6}$$

Solutions of equations (5) in closed form can be found for a propagating DW, regarding φ and θ as functions of the coordinate in a moving frame $x' = x - vt$ [25–27].

In the so-called critical regime (or 'displacive' regime, as customarily named in SPT dynamics), $\lambda \geqslant \lambda_c$, these solutions take the simple form of a polarization kink [25]:

$$S_z = \tfrac{1}{2} S_{z0} \tanh\left(\frac{\gamma}{R_0}\frac{x - x_0 \mp vt}{2}\right) \tag{7}$$

where

$$S_{z0} = [1 - 1/(4\lambda^2)]^{1/2} \simeq 2(\lambda - \lambda_c)^{1/2}$$

$$\gamma/R_0 = (\sqrt{2}/R_0)[(4\lambda^2 - 1)/(4\lambda^2 - \beta)]^{1/2} \simeq 2(\sqrt{2}/R_0)[(\lambda - \lambda_c)/(1 - \beta^2)]^{1/2}$$

denotes the inverse width of the kink centred at x_0 and $\beta = v/v_0$, $v_0 = \Omega R_0/\hbar\sqrt{2}$ being the limiting velocity of the soliton. Note that here and further $s = 1/2$ was taken.

The unperturbed IMTF possesses two integrals of motion, the energy relative to the GS,

$$\Delta E = \int \frac{\mathrm{d}x}{R_0}\left[\mathcal{H}_s(x) + \frac{J}{4}\left(1 + \frac{1}{4\lambda^2}\right)\right] \tag{8}$$

and momentum,

$$P = -\frac{\hbar}{2}\int \frac{\mathrm{d}x}{R_0} u(x,t)\frac{\partial\varphi}{\partial x}. \tag{9}$$

For the particular case of kink-like solutions (7) one has the typical relativistic-like form for the above quantities (8) and (9), e.g. [25]

$$\Delta E = \frac{J\lambda}{2\sqrt{2}}\frac{S_{z0}^3}{(1-\beta^2)^{1/2}} \qquad P = \frac{S_{z0}^3\hbar}{2R_0}\frac{\beta}{(1-\beta^2)^{1/2}}. \tag{10}$$

In the presence of perturbations, energy exchange arises between a soliton and an external subsystem. Therefore the soliton energy and momentum are no longer integrals of motion.

In the weak-coupling limit the soliton does not influence the dynamics of the vibrational subsystem significantly. Thus, phonons should be treated as an ideal gas of quasi-particles that form the thermal bath. On the other hand, changes of the internal structure of the kink can also be disregarded. Under such conditions the state of the kink will be described by the given values of its momentum and position.

4. Effective Lagrangian and equations of motion

Our prior interest is to find equations of motion for kink momentum and position. To this end we shall utilize the collective coordinate method, which consists of treating these variables as a pair of canonically conjugated ones [8, 28–30]. A strict treatment demands taking into account the influence of spin (or pseudospin) waves, which are supported by the stationary DW. These excitations are usually stated in terms of small deviations of spin components from their equilibrium values [6, 31, 32]. As excitations of the same (spin) field, the interaction between kinks and magnons (or pseudomagnons, for highly anharmonic SPT subsystems [32]) has a quadratic form in their corresponding variables (e.g. classical spin-polar angles and quantized spin deviations, respectively). Additional interaction terms, of third and fourth order in magnon operators as well as quadratic in kink variables, also arise at rather negligible scales of the order $1/\sqrt{N}$ and $1/N$, N being the number of chain sites. Although very important, such kink–magnon effects are beyond our present scope, so hereafter we restrict ourselves to the interaction of the kink with the 3D acoustic phonon field only.

Now we shall derive an effective Lagrangian of the system that does permit our dynamic subsystem (the kink DW) to be represented through its canonical variables, position $\xi(t)$ and momentum $P(t)$. The procedure consists of substituting of DW solutions (7) into the total Lagrangian of the system:

$$L = \int \frac{\mathrm{d}x}{R_0}\left(\frac{\hbar}{2}u(x,t)\frac{\partial\varphi(x,t)}{\partial t} - \mathscr{H}_{\mathrm{s}} - \mathscr{H}_{\mathrm{int}}\right) + L_{\mathrm{ph}} \tag{11}$$

where

$$L_{\mathrm{ph}} = \frac{\mathrm{i}\hbar}{2}\sum_{q}(\dot{b}_q^+ b_q - b_q^+ \dot{b}_q) - H_{\mathrm{ph}}. \tag{12}$$

Herein we have to integrate over x, keeping in mind that in the presence of a thermostat (phonons) DW parameters become time-dependent. So we introduce the *ansatz*: $u = u[x - \xi(t)]$ and $\varphi = \varphi[x - \xi(t)]$. That implies $\partial\varphi/\partial t = -\dot{\xi}\,\partial\varphi/\partial\xi$. After integration over $z = x - \xi(t)$ we obtain the effective Lagrangian in the form:

$$L_{\mathrm{eff}} = \dot{\xi}P - \Delta E - \frac{1}{\sqrt{N}}\sum_{q} G_q\, \mathrm{e}^{\mathrm{i}q\cdot\xi}(b_q + b_{-q}^+) + L_{\mathrm{ph}}. \tag{13}$$

Here G_q designates the dressed spin–phonon interaction

$$G_q = F_q[f_1(q) + f_2(q)] \tag{14}$$

with

$$f_1(q) = \frac{S_{z0}^2}{\gamma^2}\frac{\pi(q\cdot R_0)}{\sinh[\pi(q\cdot R_0)/\gamma]}$$

$$f_2(q) = \frac{S_{z0}^2\gamma^2}{12}\frac{\pi(q\cdot R_0)}{\gamma^2}\frac{1}{\sinh[\pi(q\cdot R_0)/\gamma]}\left(1 + \frac{\pi(q\cdot R_0)}{\gamma}\right)\left[4 + \left(\frac{\pi(q\cdot R_0)}{\gamma}\right)^2\right]. \tag{15}$$

Lagrangian (13) greatly resembles the polaron one (Fröhlich polaron) provided that our 'polaron' variables P and ξ are purely classical. Sometimes hereafter we shall term L_{eff} (or corresponding Hamiltonian $H_{\mathrm{eff}} = P\dot{\xi} - L_{\mathrm{eff}}$) the 'quasi-polaron' Lagrangian (Hamiltonian), having in mind this correspondence, which is not merely formal. Namely, the soliton can cause a local distortion of the crystal lattice, which follows the motion of the DW instantaneously. This effect deserves special attention and will be discussed separately.

The pair of DW variables satisfies the Hamilton equations

$$\dot{P} = -\partial H_{\mathrm{eff}}/\partial\xi \qquad \dot{\xi} = \partial H_{\mathrm{eff}}/\partial P. \tag{16}$$

Here H_{eff} designates the quasi-polaron Hamiltonian,

$$H_{\mathrm{eff}} = \Delta E + \frac{1}{\sqrt{N}}\sum_{q} G_q\, \mathrm{e}^{\mathrm{i}q\cdot\xi}(b_q + b_{-q}^+) + H_{\mathrm{ph}}. \tag{17}$$

5. Kink kinetic equation and friction

Let us now derive the KE for the kink. We shall follow the general formalism of Zubarev [33] for the description of the dynamic system in the presence of a thermostat based on the non-equilibrium statistical operator method. This approach can be easily generalized to the present problem, where we have a classical particle (kink) interacting with a quantum-mechanical thermal bath (phonons). Here we shall briefly quote the original method of Zubarev who considered the following model (index m) Hamiltonian:

$$H_{\mathrm{m}} = \sum_i H_0(p_i, q_i) + H_{\mathrm{B}}(P_{\mathrm{B}}, Q_{\mathrm{B}}) + \sum_i W(p_i, q_i; P_{\mathrm{B}}, Q_{\mathrm{B}}). \qquad (18)$$

Here H_0 characterizes the small subsystem (soliton in our case) with dynamic variables p_i, q_i; $H_{\mathrm{B}}(P_{\mathrm{B}}, Q_{\mathrm{B}})$ is the Hamiltonian of a thermal bath (index B) with complete set of variables P_{B}, Q_{B}; and $W(p_i, q_i; P_{\mathrm{B}}, Q_{\mathrm{B}})$ is assigned to an interaction part, which is generally assumed to be weak.

The macroscopic state of the dynamic system is characterized by the distribution function defined as

$$f(p, q) = \langle n(p, q) \rangle \qquad (19)$$

where $n(p, q) = \Sigma_i\, \delta(p - p_i)\delta(q - q_i)$ is the particle density, while the symbol $\langle \ldots \rangle$ denotes an averaging over a certain non-equilibrium statistical operator. The procedure of deriving the KE does not depend on both the explicit form of H_{B} and the interaction of 'particles' with the thermal bath. Furthermore it is fully independent of the nature of the thermostat itself being classical or quantum. Upon introducing suitable re-notation of the kink variables, meaning that $z_1 = q$, $z_2 = p$, strict application of the Zubarev method leads to an equation that can be written in concise form as:

$$\frac{\partial f}{\partial t} + \sum_{k,l=1,2} (-1)^k (1 - \delta_{kl}) \frac{\partial}{\partial z_k} \Big[H_0(z_2, z_1) + \langle W(p_2, p_1; P_{\mathrm{B}}, Q_{\mathrm{B}}) \rangle \Big] \frac{\partial f}{\partial z_l}$$
$$= \sum_{k,l=1,2} (-1)^{k+1} \frac{\partial}{\partial z_k} \Big[L_{kl}(z_2, z_1) \Big(\frac{\partial H_0(z_2, z_1)}{\partial z_k}$$
$$\times f(p_2, p_1) + kT \frac{\partial f}{\partial z_k} \Big) \Big]. \qquad (20)$$

Here $L_{kl}(z_2, z_1)$ are the kinetic coefficients defined as follows:

$$L_{kl}(z_2, z_1) = \frac{1}{kT} \int_{-\infty}^{0} \mathrm{d}t\, \mathrm{e}^{\varepsilon t} \Big\langle \frac{\partial W(z_2, z_1; P_{\mathrm{B}}, Q_{\mathrm{B}})}{\partial z_k} \frac{\partial W(z_2, z_1; P_{\mathrm{B}}, Q_{\mathrm{B}})}{\partial z_l} \Big\rangle_0 \qquad (21)$$

where the symbol $\langle \ldots \rangle_0$ stands for a statistical average over the thermal bath, i.e. $\langle \ldots \rangle_0 \equiv \mathrm{Tr}\, \rho_{\mathrm{B}} \ldots$, ρ_{B} being an equilibrium statistical operator of the thermostat.

In our case $W(p, q; P_{\mathrm{B}}, Q_{\mathrm{B}})$ represents the interaction of the kink with the 3D phonon field (second term in (17)). Therefore the variables p and q have to be replaced with DW momentum (P) and position (ξ), while the variables of the thermal bath, P_{B} and Q_{B}, shoud be replaced by the phonon operators, b^+ and b, respectively. From the explicit expression of $W(P, \xi; b^+, b)$, being linear in phonon operators, it is clear that its value as averaged over the equilibrium phonon distribution is equal to zero, e.g. $\langle W(P, \xi; b^+, b \rangle_0 = 0$. Besides, taking into account that in our case $\partial H_0/\partial q \equiv \partial(\Delta E)/\partial \xi = 0$, the direct calculation gives $L_{12} = L_{21} = 0$.

For further calculations we adopt the continuum approximation, which implies $f_1 \simeq S_{z0}^2/\gamma$ and $f_2 \simeq S_{z0}^2\gamma/3$, since then $(\boldsymbol{q} \cdot \boldsymbol{R}_0) \to 0$ (cf equation (15)). As we do not know the explicit solution for $\xi(t)$, as a first approximation we shall take $\xi = x_0 + vt$, which implies $\partial u/\partial \xi \to (\partial u/\partial \xi)|_{\xi = x_0 + vt}$, v being the initial DW velocity. Such an approach also means that one can neglect the dependence of the interaction term in (13) on the instantaneous DW velocity $v_{\rm DW}$, thus regarding G_q as just depending on v, whence $\partial W/\partial p \equiv \partial W/\partial P = 0$. Straightforward calculation also gives $\partial H_0/\partial p \equiv \partial(\Delta E)/\partial P = v_{\rm DW}$. As a consequence of all the above-stated facts, KE (20) finally acquires the form of the Fokker–Planck equation,

$$\partial f(P, \xi; t)/\partial t + v_{\rm DW}\, \partial f(P, \xi; t)/\partial \xi$$

$$= \frac{\partial}{\partial P}\left[L_{22}(P, \xi)\left(v_{\rm DW} f(P, \xi; t) + kT\frac{\partial f}{\partial P}\right)\right]. \tag{22}$$

It is obvious that the kinetic coefficient L_{22} herein has the meaning of a friction constant, and has the following form:

$$L_{22} = \frac{1}{kT}\frac{1}{N}\sum_q \left(\frac{\boldsymbol{q}\cdot\boldsymbol{R}_0}{R_0}\right)^2 |G_q|^2$$

$$\times \{\bar{\nu}_q \delta[\omega_q - (\boldsymbol{q}\cdot\boldsymbol{v})] + (\bar{\nu}_q + 1)\delta[\omega_q + (\boldsymbol{q}\cdot\boldsymbol{v})]\}. \tag{23}$$

The presence of δ-functions in (23) enables one to find the explicit expression for the friction constant, assuming a simple form for the phonon dispersion: $\omega_q = c_0|\boldsymbol{q}|$ for the isotropic phonon spectrum, or $\omega_q = (c_\parallel q_\parallel^2 + c_\perp q_\perp^2)^{1/2}$ for the anisotropic one; c_0, $c_\parallel$ and $c_\perp$ are, respectively, average, longitudinal and transverse speeds of sound. A further analytical step to be done below consists of passing from summation over $\boldsymbol{q}$ to integration, adopting the rule

$$\frac{1}{N}\sum_q \to \frac{3}{2q_{\rm D}^3}\int_0^{q_{\rm D}} q^2\, {\rm d}q \int_0^{\pi} \sin\theta\, {\rm d}\theta \ldots$$

as well as

$$\frac{1}{N}\sum_q \to \frac{1}{q_\parallel^{\rm D}(q_\perp^{\rm D})^2}\int_{-q_\parallel^{\rm D}}^{q_\parallel^{\rm D}} {\rm d}q_\parallel \int_0^{q_\perp^{\rm D}} q_\perp\, {\rm d}q_\perp \ldots$$

for isotropic and anisotropic cases, respectively, index D being associated with Debye's momentum. After performing integration over θ, we obtain for the isotropic case,

$$L_{22} = \frac{B^2}{kT}\frac{3\alpha^2\hbar R_0^2 c_0^5}{4mq_{\rm D}^3 v^7}\int_0^{q_{\rm D}} q^4\, {\rm d}q(\bar{\nu}_q + \tfrac{1}{2}) \tag{24}$$

and, analogously, for the anisotropic case,

$$L_{22} = \frac{B^2}{kT}\frac{\alpha^2\hbar R_0^2 c_\perp^3}{m(q_\perp^{\rm D})^3 q_\parallel^{\rm D}(v^2 - c_\parallel^2)^{5/2}}\int_0^{q_\perp^{\rm D}} q_\perp^4\, {\rm d}q_\perp(\bar{\nu}_{q_\perp} + \tfrac{1}{2}). \tag{25}$$

In the above formulae (24) and (25), $B = S_{z0}^2(1/\gamma + \gamma/3)$, $\bar{\nu}_q$ is the ordinary mean number of phonons, provided that in the case of (25) the dispersion $\omega_q = q_\perp c_\perp v/(v^2 - c_\parallel^2)^{1/2}$ has to be used. The limit wavenumbers $q_\parallel^{\rm D}$ and $q_\perp^{\rm D}$ can be well approximated by π/R_0 and $\pi/R_\perp$, respectively, where $R_\perp$ denotes the inter-chain distance.

For isotropic phonons the asymptotic behaviour of the friction constant in the low- and high-temperature limits is predicted as follows:

$$L_{22} = \begin{cases} \dfrac{B^2\alpha^2R_0^2}{mq_D^3v^7\hbar^4}\,\theta^4 & \theta \ll \hbar c_0 q_D \\ \delta_1 + \delta_2\theta^{-1} & \theta \gg \hbar c_0 q_D \qquad \theta \equiv kT \end{cases} \tag{26}$$

with

$$\delta_1 = 3B^2\alpha^2R_0^2 q_D c_0^4/16mv^7 \qquad \delta_2 = \tfrac{2}{5}\hbar q_D c_0 \delta_1.$$

A similar result is obtained for anisotropic phonons:

$$L_{22} = \begin{cases} \dfrac{24B^2\alpha^2R_0^2}{m(q_\perp^D)^2q_\parallel^D}\dfrac{\theta^4}{\hbar^4c_\perp^2v^5} & \theta \ll \hbar c_\perp q_\perp^D \\ \delta_1' + \delta_2'\theta^{-1} & \theta \gg \hbar c_\perp q_\perp^D \end{cases} \tag{27}$$

with

$$\delta_1' = \frac{B^2\alpha^2R_0^2c_\perp^2(q_\perp^D)^2}{4mq_\parallel^D(v^2-c_\parallel^2)v} \qquad \delta_2' = \frac{\hbar c_\perp q_\perp^D v}{(v^2-c_\parallel^2)^{3/2}}\delta_1'.$$

6. Brownian motion and Cherenkov-like radiation

On the basis of the present study one can make the assertion that the dynamics of DW in the IMTF as influenced by the 3D phonon field has the character of Brownian motion. This is a consequence of Cherenkov-like radiation of acoustic waves. Namely, in compliance with (23), $L_{22} \neq 0$ only if the following condition is fulfilled:

$$\omega_q \pm vq\cos\tilde{\theta} = 0. \tag{28}$$

Here $\tilde{\theta}$ is the angle between the polarization vector of an emitted acoustic phonon and the direction of motion of the DW. It means that this process can occur only when the DW velocity (v) exceeds the phase velocity of sound, i.e. if $v \geq \omega_q/q$, thus revealing the typical Cherenkov character. Such a conclusion is quite different from those obtained in the series of papers by Ivanov, Bar'yakhtar and co-workers [8, 29, 30]. They found that Cherenkov-like radiation in some real magnetics, for instance in orthoferrite-type ferromagnets, is significantly different from that occurring under the standard conditions [34]. There it was shown that a DW excites only those phonons whose momentum is perpendicular to the plane of the DW, thus predicting the absence of typical Cherenkov cone. This conclusion was mainly a consequence of an inappropriate theoretical approach to the DW dynamics whereby both subsystems (spins and phonons) were considered as purely one-dimensional. In accordance with such an assumption the non-linear character of phonon dispersion has a special role. Namely, in the case of linear dispersion there arise singularities in the friction constant (L_{22}) when the DW velocity approaches the speed of sound. In the present context the divergences in L_{22} arise when the DW velocity tends to the longitudinal speed of sound. In turn, for the anisotropic spectrum of phonons the condition (28) should be written as $v \geq c_\parallel$.

From the theory of Brownian motion it is well known that the Fokker–Planck equation is fully equivalent to the set of Langevin equations:

$$m_s\,\partial\xi/\partial t = P \qquad \dot{v}_{DW} + (L_{22}/m_s)v_{DW} = (1/m_s)F(t). \tag{29}$$

Here $F(t) = -\mathrm{d}W/\mathrm{d}\xi$ is the 'fluctuation' force, while the effective soliton mass is defined by [25]

$$m_s = (2\sqrt{2}\,\lambda\hbar^2/JR_0^2)\left[\left(1-\frac{1}{4\lambda^2}\right)^{1/2} - (1/2\lambda)\sin^{-1}\left(1-\frac{1}{4\lambda^2}\right)^{1/2}\right]$$
$$\simeq (\sqrt{2}\,\hbar^2/6JR_0^2)S_{z0}^3. \tag{30}$$

Finally, using (29) the standard procedure can be applied to calculate the diffusion constant attributed to the motion of a kink. Upon completion the final result reads

$$D = kT/L_{22}(T). \tag{31}$$

This formula represents the well known Einstein's result adapted here to the motion of a DW that evolves in the system described by the IMTF in the presence of the 3D phonon field. As is apparent from (26) and (27), the expression (31) acquires an ordinary form just under very restrictive conditions in the limit of high temperatures.

7. Concluding remarks

Concluding this paper it should be pointed out that although we have focused our analysis on the particular case of soliton (DW) dynamics in 1D IMTF within a vibrating 3D crystal lattice, our main conclusions are quite general. They are relevant for the whole class of real substances in which 1D magnetic, ferroelectric or other highly anharmonic (structurally unstable) subsystems exist within 3D crystal lattices [35]. Therefore we expect the radiation of acoustic waves of Cherenkov type to occur in most such materials. The greater part of our results should be valid also for other models, at least qualitatively. Namely, in a recent paper we have predicted almost the same behaviour of so-called magnon solitons in the easy-axis Heisenberg model that interact with 3D phonons [36].

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